AIChE Letter: Process Systems Engineering

Real-Time Hoist Scheduling for Multistage Material Handling Process Under Uncertainties

Na Tian, Ada Che, and Jianguang Feng
School of Management, Northwestern Polytechnical University, Xi'an 710072, China

DOI 10.1002/aic.14048
Published online February 25, 2013 in Wiley Online Library (wileyonlinelibrary.com)

Significance

Zhao et al. have recently developed an interesting mixed-integer linear programming (MILP) model for real-time hoist rescheduling in a multistage electroplating line. In this letter, we reformulate the hoist movement constraints in their model. Due to this reformulation, we obtain a more compact MILP model in terms of the number of constraints and variables. Computational experiment shows that our improved model can be solved several times faster than Zhao et al.'s model. Such a reduction in computation time is significant in a real-time hoist rescheduling context.

Keywords: hoist scheduling, real-time scheduling, mixed-integer linear programming

Introduction

Zhao et al.[1] have recently developed an interesting mixed-integer linear programming (MILP) model for real-time hoist rescheduling in a multistage electroplating line. The system consists of a number of processing units and a single material handling hoist. Each job is first picked up from the loading zone, and then processed successively on the units according to its processing recipe, and finally leaves the system from the unloading zone. Different jobs may have different processing recipes. The processing time of a job on a unit should be within its corresponding time window determined by its recipe. The units can process only one job at a time except those with long processing times having multijob processing capacity. After the processing of a job in a unit has been completed, it should be first lifted up by the hoist, and then transported by the hoist to the next unit, and finally dropt in the unit.

Jobs to be processed arrive randomly at the loading zone. When one or multiple jobs arrive, an optimal hoist reschedule should be determined based on the current state of the jobs inline and the hoist. The objective is to minimize the time span required for processing all the jobs inline while satisfying the following three types of constraints:[1]

- Hoist movement constraints. The release time for the hoist to drop any job in any unit and its corresponding lift time upon completion of processing are well defined so that the hoist is not required to handle more than one job at any time.
- Unit processing capacity constraints. The number of jobs simultaneously processed in any unit cannot exceed its capacity at any time.
- Processing time constraints. The processing time of a job in a unit should be within its prescribed processing time window.

In Zhao et al.'s model, one of the hoist movement constraints was first formulated as a nonlinear inequality (9) and then it was transformed into linear ones by introducing additional variables. In this letter, we demonstrate that the constraint mentioned above can be directly formulated as linear inequalities without introducing any additional variables. We also show that some constraints in their model are redundant. Thus, we obtain a more compact MILP model in terms of the number of constraints and variables. Computational experiment shows that our improved model can be solved several times faster than Zhao et al.'s model.

Reformulation of Hoist Movement Constraints

In Zhao et al.'s model, the nonlinear inequality (9) ensures that if $\mathrm{TS}_{i,n} > \mathrm{TS}_{i',n'}$, then $\mathrm{TS}_{i,n} \geq \mathrm{TE}_{i'++1,n'} + F_{i'++1,i}$, and vice versa. Zhao et al. then transformed inequality (9) into linear inequalities (10)–(12) by introducing additional variables $\mathrm{Sl}_{i,n,i',n'}$, for $i \in SI_n^A$, $i' \in SI_{n'}^A$; $n, n' \in \mathrm{SN}$; $i \neq i'$ or $n \neq n'$. In fact, to ensure that if $\mathrm{TS}_{i,n} > \mathrm{TS}_{i',n'}$, then $\mathrm{TS}_{i,n} \geq \mathrm{TE}_{i'++1,n'} + F_{i'++1,i}$ (and vice versa), the following linear constraints are sufficient

Correspondence concerning this article should be addressed to A. Che at ache@nwpu.edu.cn.

Table 1. Computation Results for Comparison of the Two Models

Case	Model	Number of constraints	Number of variables	Optimal time span	Computation time (in CPU seconds)
Case 1	Zhao et al.'s model	2699	697	283	1.594
	Improved model	795	425	283	0.203
	Zhao/Improved	3.39	1.64	1.00	7.85
Case 2	Zhao et al.'s model	1320	347	235	0.390
	Improved model	396	215	235	0.109
	Zhao/Improved	3.33	1.61	1.00	3.58
Case 3	Zhao et al.'s model	1573	417	248	0.578
	Improved model	481	261	248	0.125
	Zhao/Improved	3.27	1.60	1.00	4.62
Case 4	Zhao et al.'s model	5361	1343	426	10.015
	Improved model	1497	791	426	1.906
	Zhao/Improved	3.58	1.70	1.00	5.25

$$\mathrm{TS}_{i,n} \geq \mathrm{TE}_{i'++1,n'} + F_{i'++1,i} - M\left(1 - w_{i,n,i',n'}\right), \forall i \in \mathrm{SI}_n^{\mathrm{A}}, \forall i' \in \mathrm{SI}_{n'}^{\mathrm{A}}; \forall n, n' \in \mathrm{SN}\,; i \neq i' \quad \text{or}\, n \neq n' \quad \text{(T-1)}$$

$$w_{i,n,i',n'} + w_{i',n',i,n} = 1, \forall i \in \mathrm{SI}_n^{\mathrm{A}}, \forall i' \in \mathrm{SI}_{n'}^{\mathrm{A}}; \forall n, n' \in \mathrm{SN}; i \neq i' \quad \text{or}\, n \neq n' \quad \text{(T-2)}$$

To distinguish the constraints given in this letter and those in Zhao et al.'s model, the former are prefixed with a T. Inequality (T-1) ensures that if $\mathrm{TS}_{i,n} > \mathrm{TS}_{i',n'}$ (i.e., $w_{i,n,i',n'} = 1$), then $\mathrm{TS}_{i,n} \geq \mathrm{TE}_{i'++1,n'} + F_{i'++1,i}$, whereas equality (T-2) ensures the consistency of variables $w_{i,n,i',n'}$ and $w_{i',n',i,n}$. With constraints (T-1) and (T-2), we now show that if $w_{i,n,i',n'} = 0$, then $\mathrm{TS}_{i',n'} \geq \mathrm{TE}_{i++1,n} + F_{i++1,i'}$ must hold. If $w_{i,n,i',n'} = 0$, then it follows from (T-2) that $w_{i',n',i,n} = 1$. Consequently, we have from (T-1) that $\mathrm{TS}_{i',n'} \geq \mathrm{TE}_{i++1,n} + F_{i++1,i'}$. Thus, if $w_{i,n,i',n'} = 0$, then $\mathrm{TS}_{i',n'} \geq \mathrm{TE}_{i++1,n} + F_{i++1,i'}$ must hold. From the above analysis, constraints (T-1) and (T-2) ensure that if $\mathrm{TS}_{i,n} > \mathrm{TS}_{i',n'}$, then $\mathrm{TS}_{i,n} \geq \mathrm{TE}_{i'++1,n'} + F_{i'++1,i}$, and vice versa. We see that constraints (T-1) and (T-2) are linear. Thus, it is no longer necessary to use the nonlinear inequality (9) and then transform it into linear inequalities (10)–(12) by introducing additional variables $\mathrm{Sl}_{i,n,i',n'}$. We can find that constraints (T-2) and (T-2) are more compact than inequalities (10)–(12) in Zhao et al.'s model in terms of the number of constraints and variables. We note that constraints (T-1) and (T-2) are a variant of the hoist movement constraints for the single job-type cyclic hoist scheduling in the literature.[2–5]

We now show that due to constraints (T-1) and (T-2), inequalities (7) and (8) in Zhao et al.'s model are also unnecessary and can be removed from the model. We first discuss the case when $w_{i,n,i',n'} = 1$. If $w_{i,n,i',n'} = 1$, inequalities (7) and (8) in Zhao et al.'s model ensure that

$$\mathrm{TE}_{i++1,n} \geq \mathrm{TS}_{i',n'}, \forall i \in \mathrm{SI}_n^{\mathrm{A}}, \forall i' \in \mathrm{SI}_{n'}^{\mathrm{A}}; \forall n, n' \in \mathrm{SN}\,; i \neq i' \quad \text{or}\, n \neq n' \quad \text{(T-3)}$$

$$\mathrm{TS}_{i,n} \geq \mathrm{TE}_{i'++1,n'}, \forall i \in \mathrm{SI}_n^{\mathrm{A}}, \forall i' \in \mathrm{SI}_{n'}^{\mathrm{A}}; \forall n, n' \in \mathrm{SN}\,; i \neq i' \quad \text{or}\, n \neq n' \quad \text{(T-4)}$$

Note that if $w_{i,n,i',n'} = 1$, it follows from inequality (T-1) that

$$\mathrm{TS}_{i,n} \geq \mathrm{TE}_{i'++1,n'} + F_{i'++1,i}, \forall i \in \mathrm{SI}_n^{\mathrm{A}}, \forall i' \in \mathrm{SI}_{n'}^{\mathrm{A}}; \forall n, n' \in \mathrm{SN}; i \neq i' \quad \text{or}\, n \neq n' \quad \text{(T-5)}$$

We can find that inequality (T-5) is tighter than inequalities (T-4) and (T-3) because $\mathrm{TE}_{i++1,n} = \mathrm{TS}_{i,n} + L_{i,n}$ and $\mathrm{TE}_{i'++1,n'} = \mathrm{TS}_{i',n'} + L_{i',n'}$. To summarize, if $w_{i,n,i',n'} = 1$, inequality (T-1) is tighter than inequalities (7) and (8).

We now discuss the case when $w_{i,n,i',n'} = 0$. If $w_{i,n,i',n'} = 0$, inequalities (7) and (8) in Zhao et al.'s model mean that

$$\mathrm{TS}_{i',n'} \geq \mathrm{TE}_{i++1,n}, \forall i \in \mathrm{SI}_n^{\mathrm{A}}, \forall i' \in \mathrm{SI}_{n'}^{\mathrm{A}}; \forall n, n' \in \mathrm{SN}; i \neq i' \quad \text{or}\, n \neq n' \quad \text{(T-6)}$$

$$\mathrm{TE}_{i'++1,n'} \geq \mathrm{TS}_{i,n}, \forall i \in \mathrm{SI}_n^{\mathrm{A}}, \forall i' \in \mathrm{SI}_{n'}^{\mathrm{A}}; \forall n, n' \in \mathrm{SN}\,; i \neq i' \quad \text{or}\, n \neq n' \quad \text{(T-7)}$$

Similarly, in this case, we can also show that inequality (T-1) is tighter than inequalities (T-6) and (T-7). To summarize, inequality (T-1) is tighter than inequalities (7) and (8) in both cases and the latter ones can be removed from the model. The model becomes more compact in terms of the number of constraints.

To sum up, hoist movement constraints (7), (8), and (10)–(12) in Zhao et al.'s model can be replaced with constraints (T-1) and (T-2). Due to this replacement, the improved model becomes more compact in terms of the number of constraints and variables. This may greatly reduce its solution time.

Furthermore, it is better to replace the "$\leq$" in constraint (17) of Zhao et al.'s model with "$=$". It ensures that: (1) if $x_{i,n,n'} = 1$, then $v_{i,n,n'} = 1$ and $u_{i,n,n'} = 1$ (i.e., $\mathrm{TE}_{i,n} \leq \mathrm{TE}_{i,n'} \leq \mathrm{TS}_{i,n}$); (2) if $v_{i,n,n'} = 1$ and $u_{i,n,n'} = 1$, then $x_{i,n,n'} = 1$. Note that if we replace the iff "$\Longleftrightarrow$" in logic constraint (13) with "$\Rightarrow$", the optimal objective value remains unchanged since such a replacement would not lead to a tighter problem. It can also be shown that logic constraint (13) with "$\Rightarrow$" can be equivalently converted to linear constraint (17) with "$\leq$". Thus, both formulations with "$\leq$" and "$=$" in constraint (17) should lead to the same optimal objective value for the problem. However, it is better to use "$=$", instead of "$\leq$", in constraint (17) to ensure the value of $x_{i,n,n'}$ be well defined as mentioned above.

Computational Results

To verify our improvement, we conduct a comparison of computational performance between Zhao et al.'s model and our improved model by using Cases 1–4 presented in Ref. 1. The models are formulated by C++ in Visual Studio environment and solved by IBM ILOG CPLEX 12.4. All numerical experiments are performed on a HP PC with 3.0 GHZ and 2 GB RAM.

Table 1 gives the computation results of the two models for Cases 1–4. In Table 1, the improved model is obtained by replacing constraints (7), (8), (10)–(12) in Zhao et al.'s model with constraints (T-1) and (T-2) given in this letter. The rows "Zhao/Improved" give the ratios between the values of the rows "Zhao et al.'s model" and "Improved model". We can see from Table 1 that the optimal time spans by using our improved model are the same as those obtained by using Zhao et al.'s model. This verifies the correctness of the improved model. We can also see from Table 1 that the number of constraints and the number of variables of the improved model are around 3.4 and 1.6 times less than those of Zhao et al.'s model, respectively. That is to say, our improved model is more compact than Zhao et al.'s model in terms of the number of constraints and variables. Consequently, as can be seen from Table 1, the improved model can be solved from 3.5 to 7.8 times faster than Zhao et al.'s model. Such a reduction in computation time is significant in a real-time hoist rescheduling context.

Acknowledgments

The authors are grateful to two anonymous reviewers and the Associate Editor for their constructive comments, which improved the presentation of the article. This work was supported in part by the Humanities and Social Science Foundation of the Chinese Ministry of Education (12XJCZH004), the National Natural Science Foundation of China (71071129), the Natural Science Foundation of Shaanxi province, China (2012JM9001), and the Humanities, Social Sciences and Management Innovation Foundation of Northwestern Polytechnical University.

Literature Cited

1. Zhao CY, Fu J, Xu Q. Real-time hoist scheduling for multistage material handling process under uncertainties. *AIChE J*. 2013;59:465–482.
2. Phillips LW, Unger PS. Mathematical programming solution of a hoist scheduling program. *AIIE Trans*. 1976;8:219–225.
3. Liu JY, Jiang Y, Zhou ZL. Cyclic scheduling of a single hoist in extended electroplating lines: a comprehensive integer programming solution. *IIE Trans*. 2002;34:905–914.
4. Leung J, Zhang G, Yang X, Mak R, Lam K. Optimal cyclic multi-hoist scheduling: a mixed integer programming approach. *Oper Res*. 2004;52:965–976.
5. Zhou Z, Che A, Yan PY. A mixed integer programming approach for multi-cyclic robotic flowshop scheduling with time window constraints. *Appl Math Model*. 2012;36:3621–3629.

Manuscript received Nov. 15, 2012, and revision received Jan. 20, 2013.